The elements of colour II: the attributes of perceived colour

David J. C. Briggs

National Art School and University of Technology, Sydney, Australia Email: djcbriggs@gmail.com

In most educational contexts, colour is presented as having a single set of three attributes or dimensions. Just three attributes suffice to describe colour as long as we consider only a single mode of colour appearance, such as colours perceived as belonging to light-reflecting objects, where the CIEdefined attributes of hue, lightness and chroma are sufficient, as are the three attributes used in the Natural Colour System (NCS), hue, blackness and chromaticness. But other attributes come into play when we consider colours perceived as belonging to light itself, including (1) light perceived to be falling on objects, or (2) light reaching the eye, whether directly from a primary light source, or by specular reflection, diffuse reflection or transmission by objects. More than three attributes are therefore required to fully describe the appearance of illuminated objects, which involves colours in multiple modes of colour appearance. This paper provides a discussion of the main modes of colour appearance followed by illustrated explanations of the six attributes of perceived colour currently defined in the CIE International Lighting Vocabulary, hue, brightness, lightness, colourfulness, chroma and saturation, along with the NCS-defined attribute of blackness and the related attribute of brilliance. Special consideration is given to the distinctions between brightness and lightness, and between colourfulness, saturation and chroma, and to the relevance of these concepts for understanding, describing and depicting the appearance of illuminated objects. Also of special interest is the influence of chromatic intensity on brightness and lightness perception (the Helmholtz-Kohlrausch effect), which I argue is connected with blackness perception, and which must be contended with in order to determine lightness in the Munsell system and CIE L*a*b*. A second issue relating to colour attributes in colour education is that, with some exceptions, hue is usually presented using just a single hue circle or "colour wheel", very often in a form embodying historical beliefs about three "primary colours". To address this issue, the section on hue discusses different kinds of simple hue circle that emphasise different relationships among hues and provide alternative hue frameworks.

Received 6 January 2023; revised 7 May 2023; accepted 8 May 2023 Published online: 21 June 2023

Introduction

A limitation of much colour education in both the arts and the sciences today is in its treatment of the attributes of perceived colour. One issue is that colour is usually presented on the assumption that a single set of three colour attributes or dimensions suffices for all purposes. These attributes are usually listed as (1) "hue", (2) " lightness" or more commonly a synonym such as "value", "greyscale value", or "tone", and (3) a term for chromatic intensity, usually either "chroma" or "saturation", and generally meaning chroma as defined by the CIE *International Lighting Vocabulary* (ILV) [1]. Further, in art and design education these three attributes are commonly portrayed as dimensions of a misleadingly simple symmetrical colour solid such as a sphere, or otherwise "colour" is presented as something separate from the dimension of lightness, using a two-dimensional hue circle or "colour wheel". Usually only one such hue circle is presented, very often in a form embodying historical beliefs about three "primary colours".

Just three attributes suffice to describe colours as long as we consider only a single *mode of colour appearance*, such as the colours perceived as belonging to light-reflecting objects. For these colours the three CIE-defined attributes of hue, lightness, and chroma suffice to specify a colour, as do the three NCS-defined attributes of hue, blackness and chromaticness. But other attributes come into play when we wish to describe colours perceived as belonging to light itself, including (1) light perceived to be falling on objects, or (2) light reaching the eye, either directly from a primary light source, or by specular reflection, diffuse reflection or transmission by objects. More than three attributes are therefore required to describe the appearance of illuminated objects, which involves colours in multiple modes of colour appearance.

The CIE ILV recognises and defines six distinct attributes of perceived colour: *hue, brightness, lightness, colourfulness, chroma* and *saturation*. These six attributes and their definitions are based on work published in the late 1970's by Robert W. G. Hunt [2-3], and have been essentially stable from the 4th edition of the CIE ILV published in 1987 through to the current edition, published in late 2020. But despite this long period of stability of the standard nomenclature, I have observed over many years that the distinctions between some of these CIE-defined attributes are often poorly understood even among very experienced colour educators, and indeed, that these distinctions sometimes seem much easier to grasp for students who are new to the subject than for those with firmly entrenched ways of thinking about colour, who may understandably resist the idea that there are important fundamental concepts about colour that they have not up to that point understood.

Several factors could have contributed to the delayed reception of these concepts in colour education. The CIE ILV has always been expensive and relatively limited in distribution in hard copy, and its definitions only became freely available in 2012 as the online CIE e-ILV¹. In addition, the terse verbal and mathematical definitions in the ILV can be difficult to understand for non-specialists. Although this issue has been addressed with excellent verbal explanations and examples by Hunt, Fairchild [4] and Kuehni [5] in particular, it is hoped that the insights, explanations and illustrations presented here, devised by the writer during many years of teaching colour in art and design courses, will facilitate more widespread understanding of these attributes. Also discussed here are two colour attributes that are not currently defined in the ILV, the attribute of *blackness* as defined in the NCS and the related attribute of *brilliance*.

This paper includes some material previously published in online presentations and resources, especially [6-7].

Modes of colour appearance

Note 2 to the CIE ILV definition of "perceived colour"² names a range of modes of colour appearance and explains that these names "are intended to distinguish among qualitative and geometric differences of colour perceptions". The terminology of these modes is not fully standardised, however, and Note 2 purports to give only "some of the more important terms", and provides links to definitions for only some of these. As noted by [8, p. 145], any classification of these modes must be a somewhat arbitrary compromise between a small number of general modes and large number of special modes, but considerable consistency exists among published schemes.

¹ <u>https://cie.co.at/e-ilv</u>

² <u>CIE e-ILV 17-22-040</u>



Figure 1: Left: Gordon's Bay, Clovelly, Sydney. Photo: David Briggs. Right: "Acoustic 2017", a light installation by Aly Indermühle³. Photo: Aly Indermühle.

As already discussed in detail in Part One, colours "perceived as belonging to an object"⁴ are called *object colours* and are said to be seen in the *object mode* of colour appearance⁵. Like all perceived colours, object colours can be *achromatic* (lacking hue), or *chromatic* (having hue). When an object is opaque, its object colour is perceived to be located at the surface of the object, and may be called more specifically a *surface colour*⁶. *Volume colours* are defined as colours "perceived as belonging to the bulk of a translucent or transparent object, as well as colours perceived as belonging to the bulk of the atmosphere or other medium through which objects are seen. In Figure 1 (left), lighter and darker object colours are perceived indistinctly through the greenish volume colour of the seawater.

The ILV definition of an "object colour" does not distinguish whether the light from the object is perceived to be diffusely reflected or radiated, and the definition of "surface colour" specifically includes both conditions. A nonluminous colour is "perceived to belong to an area that appears to be transmitting or diffusely reflecting light as a secondary light source"8. Examples of nonluminous colours are the object colours we perceive as belonging to the rocks, vegetation, buildings and seawater in Figure 1 (left) and the wall in Figure 1 (right). A luminous colour is "perceived to belong to an area that appears to be emitting light as a primary light source, or that appears to be specularly reflecting such light"9. Luminous colours thus include colours seen in the *illuminant mode*¹⁰, that is, colours "seen as ascribed to a source of illumination" (and presumably called *illuminant colours*, a term listed without a linked definition in Note 2), as well as colours seen as belonging to the "highlights" on objects (areas specularly reflecting light from a light source to an observer). Fluorescent materials viewed in daylight represent a transitional case in that they are perceived as being illuminated rather than as sources of perceivable illumination, yet they appear brighter than normal for a light-reflecting object. The terms *fluorent* and *fluorence* refer to this perception, whether evoked by stimuli that are physically fluorescent or not. Whether an area is perceived as emitting or reflecting light depends on the context of the stimulus rather than any quality of the light stimulus itself (Figure 2).

- 4 <u>CIE e-ILV 17-22-042</u>
- ⁵ <u>CIE e-ILV 17-23-029</u>
- ⁶ <u>CIE e-ILV 17-22-043</u>
- 7 <u>CIE e-ILV 17-22-054</u>
- ⁸ <u>CIE e-ILV 17-22-046</u>
- 9 <u>CIE e-ILV 17-22-045</u>

³ <u>http://indermuhle.com.au/</u>

¹⁰ <u>CIE e-ILV 17-23-028</u>



Figure 2: Viewing the image on the left normally, that is, in constancy mode, we perceive a cube having a uniform nonluminous object colour, with achromatic illumination (white light) of varying brightness falling on its three faces. Image area C is perceived to depict a brightly illuminated light-reflecting object with a nonluminous object colour of moderate lightness. In contrast, image area D on the right is perceived to depict a light-emitting object exhibiting a strongly luminous object colour (an illuminant colour), off the scale of lightness. Viewed in proximal mode, we perceive areas A to C as patches of light of progressively higher brightness and colourfulness. We may further perceive the light from image area C to be similar in colour (hue, brightness and colourfulness) to that from area D, especially if the technique of "squinting" is employed (the image areas physically match).

Although the sky is physically a secondary light source, scattering light from the sun, it is normally conceived of in painting and computer rendering as an independent light source. David Katz [9] classified the colour of a "blue or uniformly beclouded grey sky" as *film colour*, meaning a colour that seems to be located at an indefinite distance in front of the eyes, rather than belonging to a definite surface. Other colours classified as film colours by Katz include the colours seen through the eyepiece of a spectroscope, the 'intrinsic visual grey" seen in complete darkness, the bright orange seen through illuminated, closed eyelids, and colours seen through an aperture in a screen.

An *illumination colour* is defined as a "colour perceived as belonging to the light falling on objects"¹¹. Examples include the bright achromatic colour of the daylight falling on most of the scene in Figure 1 (left), the achromatic and chromatic illumination colours perceived as belonging to the lights falling on overlapping areas of the wall in Figure 1 (right), the variably bright achromatic colour of the illumination seen in Figure 2 (left), and the dim achromatic and chromatic colours of the ambient illumination and the local illumination emanating from the orange circle respectively in Figure 2 (right).

If instead of viewing an environment in the usual way, that is, as an array of illuminated and perhaps luminous objects, we look at it with the attitude of observing the two-dimensional patchwork of light reaching our eyes, we will see colours in what has been called the *proximal mode* of visual perception [10]. This mode is alternatively called the *painter's mode*, because representational painters translate (in very varied ways) the colours of the patches of light seen in this way into the colours of their paints. Such painters often faciltate this way of looking at their subject by the technique of "squinting", that is, looking through their eyelashes with half closed eyes to better make comparisons of the light from different areas. Viewed in this way the scene is darkened and blurred, reducing detail, but also takes on the appearance of a flattened plane of patches of light at an indefinite distance from the eyes that has been classified as an example of a film colour [8, p.147].

Non-painters may find it easier to perceive the light from a scene as light if they view points in the scene through an aperture in a dark opaque screen. If the aperture is small enough to prevent the light

¹¹ <u>CIE e-ILV 17-22-051</u>

from being associated with a specific object, the perceived colour is said to be an *aperture colour*¹², seen in the *aperture mode* of colour appearance¹³. Provided that the surrounding screen appears dark enough, practically any point in a daylit scene will be perceived as a point of light.

Our perception of colour has a multilayered quality that is rarely appreciated. Viewed in proximal mode, the light reaching our eyes from the various objects in our field of view exhibits colours that can be described in terms of hue, brightness and either colourfulness or saturation. Viewed in constancy mode, the light-reflecting objects occupying the same field of view will exhibit object colours that can be described in terms of hue, lightness and chroma, and the illumination perceived to be falling on these objects will exhibit illumination colours that can be described in terms of hue, brightness. There may be even more superimposed colour perceptions if we perceive object colours through the volume colour of a transparent object or medium, or object colours through a specular reflection from their surfaces.

An *unrelated colour* is defined as a "colour perceived to belong to an area seen in isolation from other colours"¹⁴. This definition is generally taken to mean areas seen *in physical isolation from other light stimuli*, and thus to include colours seen through an aperture against a dark surround, or as an isolated light viewed in a dark room. The perceived colour of a split-field stimulus of the kind used for the colour-matching studies that form the foundation of colorimetry is an example of an unrelated colour *as long as the two halves of the field match in appearance to the observer*. A *related colour* is defined as a "colour perceived to belong to an area seen in relation to other colours"¹⁵, which would include colours perceived in virtually all other situations, including the colours of a split-field stimulus in which the two halves do *not* match.

Brightness, colourfulness and saturation

Brightness is defined in the CIE ILV as the "attribute of a visual perception according to which an area appears to emit, transmit or reflect, more or less light"¹⁶. A note to this definition reiterates that the term "brightness" is not restricted to primary light sources (the contrary, non-standard usage is sometimes encountered in the literature). *Colourfulness* is defined as the "attribute of a visual perception according to which the perceived colour of an area appears to be more or less chromatic"¹⁷. Just as brightness is the perceived intensity of the light from an area, coloufulness is the perceived *chromatic* intensity of the light.

Saturation is defined as the "colourfulness of an area judged in proportion to its brightness"¹⁸. Given that the only way for the light from an area to appear less colourful while maintaining its brightness is for it to appear more whitish, saturation may also be described as the perceived freedom from a white light component of the light from an area. That is, saturation is the perceived proportion of the chromatic component of a light out of the total of its chromatic and achromatic (coloured and white light) components.

¹³ <u>CIE e-ILV 17-22-021</u>

¹⁶ <u>CIE e-ILV 17-22-059</u>

¹² <u>CIE e-ILV 17-22-044</u>

¹⁴ <u>CIE e-ILV 17-22-048</u>

¹⁵ <u>CIE e-ILV 17-22-047</u>

¹⁷ <u>CIE e-ILV 17-22-072</u>

¹⁸ <u>CIE e-ILV 17-22-073</u>



Figure 3: Left: lights of zero, moderate and high excitation purity (see Part One, Figure 10), perceived as exhibiting zero, moderate and high saturation, plotted in a CIE xy chromaticity diagram. Middle: lights exhibiting low, moderate and high saturation respectively, each at three different degrees of intensity, plotted according to brightness and saturation. Right: the same, plotted according to brightness and colourfulness.

Note 1 to CIE e-ILV 17-22-073¹⁹ states that a light with a fixed chromaticity exhibits approximately constant saturation as its luminance (intensity) varies through a wide range, except at very high and very low extremes of brightness. Figure 3 (left) shows lights of three different chromaticities, perceived as having zero, moderate and high saturation, plotted in a CIE *xy* chromaticity diagram. Figure 3 (middle) shows these lights at three different intensities. The brightest light of the saturated orange series is not as bright as the brightest white light, and so is plotted lower. But notice that in the saturated and moderately saturated series, the intensity of colour or *colourfulness* increases as the lights get brighter. The colourfulness of these lights of uniform saturation increases in step with their brightness, as expected from the definition of saturation. When we plot these lights according to their brightness and colourfulness (Figure 3, right), lines of equal saturation radiate from the origin, and saturation is represented by the *angle* with the brightness axis.

As discussed in Part One, the amount of of light that an area emits, transmits or reflects is quantified in colorimety as *luminance*, the physical power of the light weighted wavelength-by-wavelength by the responsiveness of the human visual system (the luminous efficiency function). Since brightness is defined in the CIE ILV as our perception of this amount of light, one alternative definition of brightness that has been used is "apparent luminance, the apparent amount of light coming from a visual direction" [11]. However a very important caveat to equating brightness with perceived luminance is that judgements of brightness, depending on how they are made, can involve a component of what may be called *chromatic brightness*, a perception of brightness associated with high chromatic intensity, in addition to brightness related to luminance.

¹⁹ CIE e-ILV 17-22-073



Figure 4: Left: face graphic devised by [12] assembled into an array by Troy Sobotka [13], with the level of the minimally distinct border perceived by the writer for each hue column marked with a black dot. Middle: same array desaturated in Lab mode in Adobe Photoshop to greys of the same lightness "L" and hence the same luminance. Right: detail of five successive cells from one column.

In Figure 4, these two contributions to brightness can be distinguished as two different kinds of brightness. In each hue column in the chromatic part of the figure on the left, a fixed chromatic digital colour stimulus is shown interlocking with a series of achromatic stimuli increasing in luminance and perceived lightness from bottom to top. In each column I have marked with a black dot the level at which the border between the chromatic and achromatic stimuli appears to me to be *minimally* distinct on my screen, separating zones above and below where the two stimuli show more distinct edge contrast. At the size of the lightness steps employed in the figure, this level can be confidently determined and its location does not seem to be affected by the scale at which the array is observed, although the path of the points is more conspicuous at a reduced scale. The path of these points of minimally distinct border coincides closely with the point of equal luminance in each column, as can be seen in the greyscale version of the array, desaturated in "Lab mode" in Adobe Photoshop to the grey of the same "L" (based on CIE L^*). This observation is to be expected from the conclusion of [14] that brightness comparison based on finding the minimally distinct border yields results closely in agreement with comparison based on flicker photometry (that is, adjusting the stimuli until no flicker is observed when they are alternated very rapidly), which was the method used to determine the luminous efficiency function for the CIE standard observer.

However, at this level of minimally distinct border for each hue, marking equal *luminance-related brightness*, the chromatic stimulus would readily be described as "brighter in colour" than the equiluminant grey (e.g., the middle cells in Figure 4, right). One would expect that most observers, unless they were instructed to judge brightness by the minimally distinct border method (or were painters trained to do so!), would rate the chromatic stimulus as the brighter of the two, and observers do in fact systematically report colourful stimuli as being brighter than achromatic stimuli of the same

luminance, to varying degrees depending on hue and saturation, an effect known as the *Helmholtz-Kohlrausch phenomenon*²⁰.

Colours described as "bright" in this sense are those that have close to zero blackness in the NCS system. These colours include the "pure" chromatic colours and their tints through to white that form the uppermost layer of the Munsell and NCS colour solids. Each of these colours marks the approximate upper limit of the relative luminance of the light reflected by non-fluorescent pigmented objects for a given hue and saturation. That we can perceive distance from this limit (as blackness) implies that our visual system is somehow able to the compare the relative luminance of an area to this expected maximum for its hue and saturation (see Figure 12). In Figure 4 (right), the achromatic stimulus exhibits a degree of blackness because its luminance is only moderate compared to the limit for such stimuli (white), but the cyan stimulus exhibits approximately zero blackness because this same relative luminance is at about the upper limit for its hue and saturation. I would argue that our perception of this high luminance for its hue and saturation can account for the extra component of chromatic brightness beyond the luminance-related brightness of the cyan stimulus. The degree of disparity between luminance relative to white and luminance relative to the expected maximum for a hue and saturation increases with saturation at varying rates depending on hue, being least for hues close to yellow, in a pattern that seems at least broadly compatible with the saturation and hue dependence reported in various studies for the Helmholtz-Kohlrausch phenomenon [15].

Our perception of luminance is of course also subject to all of the factors relating to the viewing environment and the observer that influence perceived colour in general²¹. In addition, as discussed in Part One, it can be difficult to make exact comparisons of luminance between different areas of an illuminated scene, or different areas of a depiction of such a scene, because our attention is normally held by colours relating to the objects in the scene rather than the light reaching our eyes.

Recall from Part One that the chromaticity of a light is a measure of the overall balance of its spectral composition at the level of its long-, middle- and short-wavelength components, as detected by the human visual system. When light from a single light source of fixed chromaticity but varying intensity falls on an opaque object with a uniform spectral reflectance, we would expect the light reflected by the object to also be uniform in chromaticity. Note 1 to CIE e-ILV 17-22-073²² therefore suggests that the light reflected by the object should exhibit approximately constant saturation, varying only in brightness.

Figure 5 illustrates how fields of uniform hue and saturation and varying brightness are readily perceived as uniformly coloured objects under varying illumination, provided that their arrangement is consistent with this interpretation. Hue and saturation seem likely to contribute to the remarkable capability of our visual system, discussed in Part One, to disentangle object colours and illumination. Uniform chromaticity, perceived as uniform hue and saturation, provides our visual system with a clue as to which patches of the environment are likely to consist of the same material, and based on that, which variations in luminance are likely to be due to variations in illumination. (Other very important clues are provided by edge qualities, which are more likely, as in this example, to be soft for illumination boundaries and sharp for reflectance boundaries).

The dimension called "saturation" (S) in the simple but widely used digital colour space HSB is a crude predictor for *saturation relative to the maximum possible for digital colours of a given hue angle* (*H*). Sequences of digital colours having constant H and S in HSB have the same ratio of R, G and B

²⁰ <u>CIE e-ILV 17-22-066</u>

²¹ <u>CIE e-ILV 17-22-040</u>, Note 1

²² <u>CIE e-ILV 17-22-073</u>

components, and thus the same chromaticity. Such sequences, generated in Adobe Photoshop, were used to depict the orange cube in Figure 2 (left), the uniform saturation series in Figure 3, and the gradients in Figure 5.



Figure 5: Left: seven fields (including the achromatic surround) of uniform chromaticity and varying luminance are readily perceived as uniformly coloured objects under varying illumination, if the arrangement of the luminance gradients is consistent with this interpretation, as here. Middle and right: digital colours from this diagram plotted in the CIE 1931 xyY colour space, using the Color Inspector 3D²³ plugin in Image J²⁴.

Lightness and chroma

The attributes of lightness and chroma are reasonably familiar in colour education, where lightness usually goes by the name "value", "greyscale value" or "tone", and chroma usually goes by the names "chroma", "intensity" or so-called "saturation". (The next section will discuss ways to communicate the distinction made between chroma and saturation in the CIE ILV). Coloured chip sorting exercises, which have long been effective as physical resources for reinforcing the concepts of lightness and chroma [16], are now also available online [17,18].



Figure 6: Illustration to elucidate the CIE ILV definitions of lightness and chroma (see text). Areas A and B on the left physically match areas A and B on the right, and going from A to B on both sides colourfulness increases in step with brightness (i.e., the colour does not become proportionately more or less whitish), illustrating uniform saturation. On the left, area B exhibits an increase in brightness and colourfulness relative to area A that is in step with the increase in the brightness of the similarly illuminated white stripe, so that the difference can be accounted for subconsciously by a difference in illumination, and the areas are perceived to exhibit similar lightness and chroma. On the right, areas A and B are judged in relation to the same white and are perceived to differ in lightness and chroma.

²³ <u>https://imagej.net/plugins/color-inspector-3d</u>

²⁴ <u>https://imagej.nih.gov/ij/</u>

In the CIE ILV, lightness is defined as the "brightness of an area judged relative to the brightness of a similarly illuminated area that appears to be white or highly transmitting"²⁵, and chroma is defined as the "colourfulness of an area judged as a proportion of the brightness of a similarly illuminated area that appears grey, white or highly transmitting"²⁶. Figure 6 may help to elucidate the idea behind these definitions. Since the brightness of an opaque object relative to the brightness of a similarly illuminated white object is in effect our perception of how much of the light falling on the object is reflected, lightness has also been defined as "apparent reflectance" [11]. Chroma can similarly be considered to be our perception of the efficiency of an object as a *spectrally selective* reflector.

Notice that unlike the definitions of brightness and colourfulness, and thus indirectly of saturation, which refer only to the appearance of an area, the definitions of lightness and chroma both refer to a similarly illuminated area. By these definitions the attributes of lightness and chroma apply only to related colours, and further, only to related colours perceived as belonging to illuminated areas, rather than to luminous objects or to light itself. Colours perceived as belonging to luminous objects or to light itself, including colours perceived in the illuminant, illumination, and proximal modes, exhibit brightness but generally seem to be off the scale of lightness. Thus, looking at stars in the sky or at city lights, we normally do not perceive the brightest light as white and all the others as shades of grey, and when we see a fall-off of illumination with increasing distance from a light source (Figure 1, right), we see the illumination as becoming dimmer (lower in brightness) but not darker grey (lower in greyscale value). (I am speaking here of the colours perceived as belonging to the light itself, not colours relating to the image surface). Perceptions of lightness can arise in related colour stimuli as simple as the two halves of a split-field stimulus that do not match [19], but the crucial factor seems to be whether the stimulus is *perceived* as an illuminated or a luminous object, rather than whether it is physically reflecting or emitting light. Thus when we look at a computer screen we readily see different areas as exhibiting lightness (greyscale value), even though the screen is physically emitting light, because we normally "read" the screen as an illuminated page rather than as a primary light source. But if an area on a screen is perceived to depict a luminous object, that virtual object seems to be off the scale of lightness (e.g. D in Figure 2, right).

Fluorescent materials viewed in daylight are normally perceived as being illuminated and can therefore exhibit lightness despite their brighter than normal appearance for a light-reflecting object, though Evans [19, p.100] found that for fluorent (fluorescent appearing) areas, lightness becomes increasingly difficult to recognise as a separate perception as fluorence increases.

Based on the CIE definition of lightness given above, we might expect the lightness of an opaque object to be related to the psychophysical quantity of its luminance factor, the ratio of the luminance of the light it reflects to the luminance of the light reflected by a perfect white object under the same illumination²⁷. The two main lightness scales familiar to artists are in fact both based on this luminance factor, though transformed in a nonlinear way to approximately equalise the perceived contrast between adjacent steps on the scale. These scales are CIE 1976 lightness, L^* (implemented as the "L" in Lab space in Adobe Photoshop), described in the CIE ILV as an "approximate correlate" of lightness²⁸, and Munsell value, which is defined such that all Munsell chips of the same value reflect light of the same luminance under Illuminant C, a standard daylight illuminant used in 1943 when the modern Munsell system was defined. The nonlinear transformations applied to the luminance factor are different in the

²⁵ <u>CIE e-ILV 17-22-063</u>

²⁶ <u>CIE e-ILV 17-22-074</u>

²⁷ <u>CIE e-ILV 17-24-077</u>

²⁸ CIE e-ILV 17-23-076

two systems but similar in effect, so that the middle step of the scale ($L^* = 50$; Munsell value = 5) has a luminance factor of about 20%.



Figure 7: Left: Munsell 10R hue page, showing notations represented by chips in the Munsell Book of Color, glossy edition. Right: CIE 1976 a, b hue angle 48° page based on CIE L*a*b*, showing notations represented in sRGB space. The grey swatch from the next highest row has been juxtaposed with the highest chroma swatch on the value 5 and L* 50 rows.

Since both Munsell value and CIE L^* take account only of luminance-related brightness and not any influence of the chromatic brightness discussed previously, in making judgments of lightness in either system one should place the sample immediately adjacent to the greyscale swatches and find the swatch that has the minimally distinct border. If we examine a horizontal row of swatches in either system, for example the Munsell value 5 row in Figure 7 (left), then as we pass outwards along this row the luminance stays the same, but the swatches appear progressively "brighter in colour" in step with their decreasing blackness, and might be judged to correspond in brightness to a grey one or more steps higher on the lightness scale. Nevertheless, it can be seen that these higher greys show a stronger edge contrast with the grey background than any swatch in the value 5 row, the perceptual sign of their higher luminance. Thus, if we examine the border between a value 6 or $L^* = 60$ grey swatch and the highest chroma swatch of the row below, we'll see that the grey swatch is the higher in luminance-related brightness of the two (i.e. it would need to be darkened to reach the minimally distinct border), even though the chromatic swatch has a relatively "glowing" appearance. Students generally succeed either immediately or with a little practice in sorting colour chips according to luminance-related lightness by undertaking the sorting exercises mentioned above.

The object-colour attributes of lightness and chroma exhibit a *tendency* towards stability, called *colour constancy*, under varying viewing conditions, especially varying intensities of the same illumination. Indeed, this tendency towards stability is the reason why these attributes are perceived as *belonging* to an object. In contrast, differently illuminated areas of a uniform object may vary greatly in brightness and colourfulness (e.g. areas A to C, Figure 2, left), but this varying brightness and colourfulness is perceived as being imposed by the illumination rather than as belonging to the object itself, and so these attributes do not form part of the object colour.

The relationship between saturation and chroma

The word "saturation" is often used as a synonym for "chroma" in colour education, but the distinction between the two attributes as defined by the CIE ILV can be easily demonstrated on a Munsell or CIE $L^*a^*b^*$ hue page (Figure 8). A vertical column of digital swatches of uniform chroma can be seen to range from lighter swatches emitting a large amount of relatively whitish light above to darker swatches emitting a smaller amount of less whitish/ more saturated light below (Figure 8, left). In contrast to these *vertical* lines of uniform chroma, swatches exhibiting similar saturation lie along lines that *radiate* from a point *near* the zero point on the value scale. Thus swatches A, B and C all exhibit similarly high saturation, emitting similarly pure orange light, but vary considerably in chroma (Figure 8, middle). Ascending along these radiating lines of uniform saturation, the light emitted by the swatches increases in colourfulness in step with its brightness, becoming neither more nor less whitish.



Figure 8: Left: swatches in a column from a Munsell hue page exhibit uniform chroma but increasing saturation going down the column. Middle: swatches exhibiting uniform saturation, including those similar to image areas A, B and C in the image of the cube to the right, are arranged along lines that radiate from near (actually one value step below) zero on the value scale. Right: areas A to C are depicted using digital colours of uniform hue angle H and "saturation" S in the colour space HSB, used in the colour picker from Adobe Photoshop.

Such lines are relevant to many painters because they show the series of paint colours, sometimes called a *shadow* or *shading* series, that could be used to depict the light of uniform chromaticity that would be reflected to the eye by a uniformly coloured opaque object under different levels of the same illumination. These lines of uniform chromaticity and saturation, which we saw were vertical in CIE xyY space (Figure 5), radiate from near the zero point on the lightness axis in hue-lightness-chroma spaces like Munsell and CIE $L^*a^*b^*$.

We might expect that along these lines, to represent colourfulness increasing in step with brightness, chroma should increase in direct proportion to lightness, and so the lines should radiate from the zero point on the lightness scale. In both the Munsell system and CIE $L^*a^*b^*$ this is approximately but not exactly true. Paul Centore [20] showed that lines of unform chromaticity projected onto Munsell hue planes drift and curve somewhat, but that straight lines fitted to them on average seem to radiate from a point about one value step below zero on the value scale. In CIE $L^*a^*b^*$, lines of uniform chromaticity

are nearly straight over most of their length, showing an apparent convergence towards a point 16 steps below zero on the 100-level L^* scale, but bend abruptly to reach the origin at very low lightness levels (Figure 9). This bend is connected with the formula by which L^* is calculated, which compresses very low luminances into a short interval at the bottom of the lightness scale. In some other colour spaces such as YCbCr, lines of uniform chromaticity radiate regularly from the origin.



Figure 9: Left: variation on diagram in Figure 5, left, in which luminances continue down to zero. Middle and right: digital colours from this diagram plotted in CIE L*a*b*, using the Color Inspector 3D²⁹ plugin in Image J³⁰.

Like lightness and chroma, saturation exhibits a degree of constancy for a given object under a wide range of intensities of illumination^{31,32}, and so can also be perceived as belonging to an object and be used as an alternative to chroma to describe a different aspect of the chromatic intensity of an object colour.

Blackness and brilliance

An entirely different way in which we can classify object colours is according to their degree of resemblance to a pure black, a pure white and a pure chromatic colour. This is the basis of the Scandinavian Natural Colour System (NCS), in which these resemblances are taken to add up to 100, allowing them to be plotted on a triangular hue page (Figure 10, left). Arranging the triangles for every hue around a central neutral axis results in a symmetrical framework in the form of a double cone, though the physical coloured chips in the NCS Atlas do not fill the triangles and form an irregular solid within the double cone. Colours are specified in the NCS according to their blackness (their degree of resemblance to a pure black [21] or "the perceived amount of black in the colour relative to pure black" [22]) and their *chromaticness* (their degree of resemblance to a pure black [21] or "the perceived amount of black in the colour [21]). In standard CIE terms, chromaticness is chroma relative to the maximum chroma perceived to be possible for the hue, but blackness is a completely distinct attribute of perceived colour not currently treated in the CIE ILV.

²⁹ <u>https://imagej.net/plugins/color-inspector-3d</u>

³⁰ <u>https://imagej.nih.gov/ij/</u>

³¹ <u>CIE e-ILV 17-22-073, Note 1</u>

³² <u>CIE e-ILV 17-22-074, Note 1</u>



Figure 10: Left: Red (R), yellow (Y), green (G) and blue (B) hue pages from the NCS Digital Atlas 1950. Right: diagrammatic subdivision of a hue page according to blackness and chromaticness (degree of resemblance to pure black and to pure chromatic colour respectively). Evans suggested that brilliance may be considered negative for blackness (his "grayness") and positive for fluorent and illuminant colours, as here, or as simply continuous from the black point [19, p.100].

Brilliance is a related scale of colour appearance investigated in detail by Evans [19, pp.99-101]. Evans showed that if a light stimulus of fixed chromaticity increases in luminance from zero to very high in relation to its surroundings, its appearance passes through a consistent series of stages, beginning with a black that shows no visible change until a threshold is reached (called the *black point* by Evans), then passing through degrees of decreasing *blackness* (called *grayness* by Evans) to a point of *zero blackness* (called *zero grayness* or G_o by Evans) and then continuing on to exhibit *fluorence* (perceived fluorescence). Evans found that perception of zero blackness occurs at a luminance that varies greatly depending on the hue and saturation of the stimulus, decreasing with increasing saturation, but at different rates for different hues. As the luminance continues to increase up to and somewhat beyond the point of matching an area perceived to be white, fluorence continues to increase, after which further increase in luminance results in the stimulus taking on the appearance of a light source (in CIE terms, an *illuminant colour*).

The circle on the right face of the cube depicted in Figure 11 illustrates four stages of this sequence. At low luminance the circle exhibits a high degree of blackness (top left). At a certain luminance relative to their white-appearing surround, the circles on all three faces exhibit approximately zero blackness (top right). At a somewhat higher luminance (bottom left), the circle on the right face exhibits fluorence. The corresponding image area here matches that depicting the circle on the top plane, but as this plane is perceived to be more strongly illuminated, that circle does not appear fluorent. At a much higher luminance *relative to the environment* (bottom right) the circle on the right face appears highly luminous, even though the image area is unchanged from the preceding image. These relationships are clearly important for painters concerned with creating the appearance of luminosity and consistent illumination, though they appear to be understood mainly intuitively at present.



Figure 11: Blackness and brilliance. The circle on the right face of each cube maintains the same chromaticity (perceived as hue and saturation) and varies in luminance relative to its environment, following the stages of blackness and brilliance shown in Figure 10, right. The perception of luminosity in the lower right image is assisted by depicting illumination derived from the circle on the nearby floor.



Figure 12: Munsell principal and intermediate hue pages from the Munsell Book of Color Glossy Edition, showing physical chips in relation to optimal colour limits (in white), with blue lines of uniform saturation/chromaticity depicted as radiating from one value step below value zero, and white contours of uniform NCS blackness plotted using the NCS Translation Key [23]. Lower right: depiction of near-zero-blackness colours of the lightest chips of each hue and chroma, showing variation of their lightness with hue and chroma.

Figure 12 shows the relationship of NCS blackness to Munsell value and chroma for ten hue pages from the *Munsell Book of Color Glossy* Edition using a key published by the NCS giving NCS notations for Munsell chips when viewed on a light background [23]. Zero blackness is associated with varying lightness levels, tracking just above the top of the range of physically realised paint chips in both the NCS and the Munsell atlases, at lightness levels that are high for all pale colours and drop steadily for increasingly chromatic colours, though less rapidly for hues near yellow than for other hues. Assuming that the colours of most light-reflecting objects are subject to broadly similar physical restrictions to those limiting the gamut of NCS and Munsell chips, the fact that we have a perception of blackness seems to show that we have an unconscious sense of how bright an object reflecting light of any given hue and saturation is likely to appear in a given illumination, just as our perception of objects having lightness implies that we have an unconscious sense of how bright a white object is likely to appear in the same illumination.

For some comments on the historical origins of the concepts of blackness and brilliance in the writings of Ewald Hering, Wilhelm Ostwald and Arthur Pope, see [24].

Hue

We saw in Part One that hue is *the way in which we perceive* an overall direction of bias in the spectral composition of an isolated light relative to daylight, or in the spectral reflectance of an object, where "overall" means at the level of its long-, middle and short-wavelength components, as detected by the human visual system. The two-dimensional circuit of hues is imposed by our dependence on the responses of three receptor types encoded by the process of cone opponency, although exactly how the circuit of hue perceptions arises from the circuit of cone-opponent responses remains unclear.

It's hardly necessary to promote the importance of hue to students. For beginning painting students at least, hue already has their attention, and it's the task of the teacher to extend some of that attention to the other attributes of colour, beginning with lightness. However, because hue is typically presented in colour education in relation to a single hue circle, very often in a form embodying historical beliefs about three "primary colours", it may be useful to conclude this paper with a discussion of some of the alternative hue scales that may be useful in different contexts. To do so, however, we must first introduce the concepts of the four unique hues and complementarity.

Hue is defined in the CIE ILV as the "attribute of a visual perception according to which an area appears to be similar to one of the colours: red, yellow, green, and blue, or to a combination of adjacent pairs of these colours considered in a closed ring"³³. Thus, the hue of any colour is its closest match in the circuit of high-chroma colours. So, olive and brown are not hues, but they *have* a hue that is the particular degree of yellow or orange that they most closely resemble. Such comparisons can be misleading if they are not made side by side: for example, on some yellow hue pages in the Munsell and NCS atlases there are olive swatches that might be classified as varieties of green, yet when these are compared side by side with a sequence of high-chroma swatches of different hues they will be found to more closely resemble the high-chroma yellow on their hue page than any high-chroma green.

Based on its ILV definition, the most obvious way to specify hue would be in relation to a scale of gradations in a circuit of intermediate hues between red, yellow, green, and blue. Red, yellow, green, and blue are identified elsewhere in the ILV as the four unique hues, a *unique hue* being defined as a

³³ <u>CIE e -ILV 17-22-067</u>

"hue that cannot be further described by the use of hue names other than its own"³⁴. The unique hues also appear outside the CIE system as the hues of the four chromatic *elementary colours* of the NCS [21-22]. In my experience, all but a very few students agree they can imagine, between what painters call a yellow with a green "bias" and a yellow with an orange "bias", a yellow that is just yellow, and similarly for red, blue, and green, and can think of all other hues as combinations of adjacent pairs of these. It has been found in studies over the last century that the samples chosen by individuals as representative of each of the four unique hues span a very large range of hues on the Munsell hue scale [25], but the average positions reported in these studies have a tighter range of two or three Munsell hues (Figure 13, lower middle). In the NCS, a page of physical swatches is taken to embody the hue of each of the four chromatic elementary colours, allowing precise specification of hue in relation to ten gradations between each adjacent pair of these standardised reference hues.



Figure 12: Simple hue circles structured (1) around the Munsell principal and intermediate hues (upper left), (2) the four unique hues (upper middle), and (3) the traditional "primary" and "secondary" colours (upper right), with relationships of each to the Munsell hue scale below. Locations of the unique hues and the corresponding traditional "primary colours" compiled from the average hue locations reported in the studies reviewed by [25].

Note 1 to the ILV definition of "unique hue" states: "There are 4 unique hues: red, green, yellow and blue forming 2 pairs of opponent hues: red and green, yellow and blue"³⁵. This note acknowledges the concept proposed in the late 19th century by Ewald Hering that the visual system generates *three opponent signals*, one being either red or green, one being either yellow or blue, and one being either white or black. The four unique hues are very commonly presented in the context of the theory of hue opponency, but the concepts are not inextricably linked. Historically the four unique hues or elementary colours were singled out by painters and theorists long before Hering's theory of hue opponency, for

³⁴ <u>CIE e-ILV 17-22-068</u>

³⁵ CIE e-ILV 17-22-068

example as the "simple" colours of Filarete and Leonardo da Vinci in the Renaissance. Opponency of neural signals correlated with the four unique hues is currently unsupported by available evidence [26].

Complementarity as defined by the CIE ILV³⁶ refers specifically to mixing of light. For any direction of spectral bias relative to white light, now specified as a *dominant wavelength* or complementary wavelength (Figure 14; see also Part One, Figure 10), there is a family of chromaticities with an opposite direction of bias that constitute an exactly opposing chromatic stimulus. When a pair of these complementary lights are combined in the right proportion their opposite biases cancel out, and the colours of the two lights seem to mix to make white light. Complementarity in this sense is an expected consequence of Newton's "centre of gravity" principle, that the colour of an isolated light is the way in which we perceive the overall balance of its spectral composition relative to white light. Newton tested experimentally for, but did not succeed in demonstrating, complementarity in *monochromatic* lights, but he studied complementary colours in broadband lights by "decompounding" white light into various pairs of lights that between them contained the full set of spectral components [27] and by observing the colours of light reflected and transmitted respectively by thin films at the same point [28].

The term "complementary colour" is also used by artists in other senses, especially to refer to *afterimage* colours (colours of negative afterimages), to the colour of a paint used to neutralise another paint in physical mixture, or to the opposite colour in a traditional (red-yellow-blue) colour wheel. It may therefore be advisable to distinguish complementary colours in the CIE sense of opposite chromatic stimuli as *additive* complementaries.

A complication to the concept of (additive) complementary colours is that light of a given dominant wavelength can shift in hue to some degree as it becomes desaturated. Because of this effect, called the *Abney phenomenon*³⁷, most hue boundaries in a chromaticity diagram curve instead of forming straight lines radiating from the white point. This means that light of a given hue is usually complementary to lights of a small range of hues; for example, the same yellow light can be complementary to both a saturated blue and a desaturated violet light (Figure 14). This limits the precision with which pairs of hues can be said to be complementary and be represented as such on a simple hue circle.

The Munsell hue scale is intended to exhibit perceptually even spacing, and is structured around five *principal hues*, a red, yellow and green (R, Y and G, each essentially representative of the corresponding unique hue), a cyan blue (B), and a purple (P). Opposite each of these are five *intermediate hues*, respectively BG (turquoise), PB (the hue of cobalt blue, widely deemed by painters to be middle blue), RP (corresponding to the magenta of painters), YR (orange) and GY (yellow-green). In the *Munsell Book of Color* each of these ten hues is represented by four hue steps, for example 2.5R, 5R (the typical red), 7.5R and 10R, making a total of 40 hue pages, as in the NCS. In addition to being perceptually evenly spaced, opposite hues in the Munsell system are additive complementaries or reasonably close (mostly within one hue step), allowing for the limited precision of the concept of complementary hue pairs stemming from the Abney phenomenon (Figure 14, right).

For many educational and practical purposes, a 40-hue scale like that of the Munsell system or the NCS is unnecessarily detailed. For example, a simple hue circle based on the four unique hues (Figure 13, upper middle) will suffice to lay out a set of chromatic paints in consistent hue order, as will a simple circle based on the five Munsell principal hues. If the intermediate hues are added the simple Munsell circle serves as a reminder of five additive complementary pairs, red/blue-green, yellow/cobalt blue, green/magenta, cyan blue/orange and purple/yellow-green. In being evenly spaced perceptually and

³⁶ CIE e-ILV 17-23-013

^{37 &}lt;u>CIE e-ILV 17-22-070</u>



showing additive complementaries, the simple Munsell circle arguably has advantages over simpler hue circles for planning hue relationships.

Figure 14: Abney phenomenon. Left: when a light is desaturated by mixture with a white or a complementary light, there may be a small shift in hue, implied by the curvature of the boundaries separating lights of different hues on a chromaticity diagram [after 29]. For example, a saturated deep blue light passes into a pale violet (top right); both lights are complementary to the same greenish yellow light. Lower right: lines of equal hue and chroma for Munsell notations of value 5 [exported from an interactive online app formerly available at 30].

The hold of the traditional "colour wheel" structured around red, yellow and blue remains strong in secondary and some tertiary education and in texts and online resources for artists. The appeal of this system can be understood if we examine paint mixing with the widespread but mistaken assumption that colours reside and mix in our paints themselves, and if we recall that the three so-called "primary colours" correspond to three of the four unique hues or elementary colours that we *perceive as* components of other hues. With this mindset we find that we indeed can't mix a red paint without using a paint that already "contains red" (e.g., magenta), and the same for yellow and blue, but we *can* mix a green paint from a blue and yellow paint that don't "contain green" (i.e., that don't appear at all greenish), which seems to prove that the colour green is physically composed of yellow and blue. This latter view prevailed in science until it was overturned in 1852 when Helmholtz explained how subtractive mixture works, but it persists today as the defining tenet of traditional RYB colour theory. (We in fact get green because blue and yellow paints both reflect some wavelengths in the green range, and it's these shared wavelengths that survive subtractive mixture).

Omitting green as a primary colour and spacing middle red, middle yellow and middle blue at equal intervals results in a hue circle that is perceptually very uneven and inevitably distorts hue relationships across the circle (Figure 13, lower right). Despite these substantial drawbacks, some very skilled painters continue to favour the traditional RYB colour wheel as a simple practical framework, although one suspects that an equally simple but less problematic framework might serve their purposes better. Perhaps the most unfortunate result of the conceptual framework of red, yellow and blue "primary colours" is that the misconceptions about colour and paint mixing that it embodies make a very poor foundation for the study of colour itself.



Figure 15: Left: a hue circle structured around yellow, magenta and blue-green provides a simple, evenly spaced hue framework that aligns with a marked difference in pattern of mixing paths for paints close to and remote from these hues. Upper right: mixing paths for six selected Liquitex acrylic paints with 18 other such paints (labelled at [31, Figure 6.3.6]) modelled using the free program drop2color by Zsolt Kovacs³⁸. Contrast the pattern of mixing paths for paints close to the reference hues with those of paints far from these hues. Lower right: representative mixing paths for paints close to and far from the three reference hues, and for a traditional "red-yellow-blue" palette.

If a three-fold system is desired, a middle yellow around 5Y, a magenta around 5RP and a blue-green around 10B provide a hue framework that's about even in perceptual spacing and also acknowledges a marked difference in patterns of mixing paths between paints close to these hues and paints remote from them (Figure 15). The former make high-chroma mixtures with paints of distant hues and are directly neutralised by paints of a narrow range of opposite hues, whereas paints distant from these hues (orange-red, yellow-green and violet blue) make high-chroma mixtures only with paints quite close in hue, and are neutralised by paints of a wide range of opposing hues. As a result of these patterns, paints of these three hues yield a relatively large gamut of physical mixtures, larger and more even than that of a traditional RYB palette. But this is really a side issue, since we can employ any palette we want with any hue circle as a conceptual framework, and to overemphasise this difference by calling cyan, magenta and yellow the "real", "modern" or "scientific" primaries risks simply transferring the misconception that all colours are "made of" three primary colours to a different set of hues.

In most graphics programs hue is measured by hue angle (H) in HSB colour space, unfortunately a very perceptually uneven hue scale in which the same angular change corresponds to a much smaller hue change in some sectors than others (Figure 16). Opposite hues are complementary on the 0°-180°, 60° -240° and 120°-300° axes, but not on intermediate axes [32]. Better than this very crude correlative of hue is hue angle measured in CIE $L^*a^*b^*$, available in the colour picker of the open-source graphics

³⁸ available at <u>https://zsolt-kovacs.unibs.it/colormixingtools/cmt-drop2color</u>

program GIMP, and hue angle measured in HSLuv, a symmetrical adaptation of CIE $L^*u^*v^*$ by Alexei Boronine [33], available in the commercial painting program *Rebelle* (version 5 onwards)³⁹. Colour hue scales for digital artists that are even more perceptually even are on the horizon including Oklab of Björn Ottosson [34] and an implementation of hue angle in CAM16 in Google's HCT space [35].



Figure 16: Left: simple hue circle based on symmetrically arranged "red", "green" and "blue" digital primaries and their binary mixtures. Middle: hue angle (H) in the widely used colour spaces HSB and HLS. Right: approximate locations of colours from Figure 16, left, and 10° increments of H from Figure 16, middle, in relation to the Munsell hue scale.

Acknowledgements

I am very grateful to my students and colleagues over many years for their feedback on my evolving attempts to explain the concepts discussed in these two papers. I'd like to especially thank Robert Hirschler, Barry Maund, Mohan Matthen, and Derek Brown for providing comments on an earlier version of Part One, as well as Professor Mark Fairchild who very kindly agreed to provide comments on the current versions of both papers. All of these provided comments that led to corrections or clarifications, but responsibility for any remaining errors is my own. I am also grateful to Troy Sobotka for permission to adapt and include a graphic he assembled to convey a somewhat different point (Part Two, Fig. 4, left).

References

- Commission Internationale de L'Eclairage (2020), CIE S 017:2020 ILV: International Lighting Vocabulary, 2nd edition. [https://cie.co.at/e-ilv – last accessed 31 May 2023]
- Hunt RWG (1977), The specification of colour appearance. I. Concepts and terms, Color Research and Application, 2 (2), 55-68.
- 3. Hunt RWG (1978), Colour terminology, Color Research and Application, 3 (2), 79-87.
- 4. Fairchild MD (2013), Color Appearance Models, 3rd edition, Chichester: John Wiley and Sons.
- 5. Kuehni RG (2013), Color: An Introduction to Practice and Principles, 3rd edition, Hoboken: Wiley.
- 6. Briggs DJC (2021), More than three dimensions: Communicating the attributes of colour perception in colour education, *Proceedings of the 14th Congress of the International Colour Association: AIC 2021*, 1073-1078, Milan, Italy.

³⁹ <u>https://www.escapemotions.com/products/rebelle/about</u>

- 7. Briggs DJC (2017), The Dimensions of Colour, Part One, The Dimensions Introduced. [http://www.huevaluechroma.com/011.php – last accessed 7 June 2023]
- 8. Committee on Colorimetry of the Optical Society of America (1953), The Science of Color, Thomas Y. Crowell Co.
- 9. Katz D (1935), The World of Colour, MacLeod RB (transl.), London: Kegan Paul, Trench, Trübner & Co.
- 10. Rock I (1983), Logic of Perception, Cambridge: MIT Press.
- 11. Arend WE (1993), Mesopic lightness, brightness and brightness contrast, Perception & Psychophysics, 54 (4), 469-476.
- 12. Kindlmann G, Reinhard E and Creem S (2002), Face-based luminance matching for perceptual colormap generation, *IEEE Visualization: VIS 2022*, 299-306, Boston, MA, USA.
- 13. Sobotka T (2022) https://twitter.com/troy_s/status/1555710038521524224 last accessed 7 June 2023
- Wagner G and Boynton RM (1972), Comparison of four methods of heterochromatic photometry, *Journal of the Optical Society of America*, 62 (12), 1508-1515.
- 15. Donofrio RL (2011), The Helmholtz–Kohlrausch effect, Journal of the Society for Information Display, 19 (10), 658-664.
- 16. Reed R (2021), The New Munsell Student Color Set, 6th edition, London: Bloomsbury Publishing.
- 17. Westland S (2018), Interactive Colour Demos by Stephen Westland and Marjan Vazirian. [http://stephenwestland.co.uk/app/demos.htm – last accessed 7 June 2023]
- 18. Dahl M (2015), Color Practice. [https://www.color-practice.com/challenge.html last accessed 7 June 2023]
- 19. Evans RM (1974), The Perception of Colour, New York: Wiley-Interscience.
- 20. Centore P (2011), Shadow series in the Munsell system, Color Research and Application, 38 (1), 58-64.
- Hård A, Sivik L and Tonnquist G (1996), NCS, Natural Color System from concept to research and applications. Part I, Color Research and Application, 21 (3), 180-205.
- 22. NCS UK (2023), NCS Glossary. [https://www.ncscolour.co.uk/information/ncs-glossary.html last accessed 7 January 2023]
- 23. NCS (2008), NCS translation key, NCS Munsell, Edition 2.
- Briggs DJC (2017), The Dimensions of Colour, Blackness and Brilliance. [http://www.huevaluechroma.com/018.php last accessed 7 June 2023].
- 25. Kuehni RG (2014), Unique hues and their stimuli-start of the art, Color Research and Application, 39 (3), 279-287.
- 26. Webster MA, Miyahara E, Malkoc G and Raker VE (2000), Variations in normal color vision. II. Unique hues, *Journal of the Optical Society of America A*, **17** (9), 1545-1555.
- Newton I (1673), An extract of Mr. Isaac Newton's letter, written to the published from Cambridge April 3.1673, Philosophical Transactions of the Royal Society of London, 8 (97), 6108-6112.
 [https://royalsocietypublishing.org/doi/epdf/10.1098/rstl.1673.0041 – last accessed 7 June 2023]
- 28. Newton I (1704), Opticks: or, a Treatise of the Reflexions, Refractions, Inflexions and Colours of Light, Book Two, Part One, London: Printed for S. Smith, and B. Walford.
- 29. Kelly KL (1943), Color designations for lights, Journal of the Optical Society of America, 33 (11), 627-632.
- 30. Bruce Lindbloom http://www.brucelindbloom.com/index.html last accessed 7 June 2023
- 31. Briggs DJC (2012), Paint Mixing Principles. [http://www.huevaluechroma.com/063.php#hue last accessed 7 June 2023]
- 32. Briggs DJC (2012), Additive Complementaries. [http://www.huevaluechroma.com/043.php last accessed 7 June 2023]
- 33. Boronine A (2022), HSLuv. [https://www.hsluv.org/ last accessed 7 June 2023]
- Ottosson B (2020), A Perceptual Color Space for Image Processing. [https://bottosson.github.io/posts/oklab/ last accessed
 7 June 2023]
- O'Leary J (2022), The Science of Color & Design. [https://material.io/blog/science-of-color-design last accessed 7 June 2023]