Instrumental colour mixing to guide oil paint artists

Eric Kirchner

Color Technology Group, AkzoNobel Paints & Coatings, 2170 BA Sassenheim, the Netherlands
Email: eric.kirchner@akzonobel.com

Painters use a creative but time-consuming process to mix differently coloured pigment paints in order to obtain an intended colour. During the past decades it became technically possible to achieve this task also using instruments. However, their high prices have prevented painters from making this choice. Very recently, a new generation of colour instruments became available at much lower prices, making this an affordable approach for both professional and amateur painters. We selected one such instrument, and tested its ability to help painters in mixing pigment paints to achieve a desired colour. Our approach started by choosing a limited set of pigmented oil paints from an artist line of a commercial brand (Van Gogh line from Royal Talens). We applied these paints in a range of well-chosen mixtures to determine the spectral properties of each pigment paint. Using a spreadsheet program and an optical model we were then able to create any desired colour by calculating the optimum mixture of pigment paints. In this article, we test the accuracy of this method for use by artist painters.

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Introduction

A basic understanding of colour theory is fundamental to painting. But traditional colour theories offer little guidance on creating the intended colours [1], and the artist's colour wheel seems to hardly capture the complexities of paint colour mixing [2-3]. Therefore it is not surprising that artists and art students often find it difficult to use colour theory in practice when they need to mix paints [4]. Mixing colours on the palette often leads to surprises that are sometimes inspirational, and sometimes frustrating. Even some experienced painters need considerable time for mixing pigment paints and testing the result before they obtain the intended colours [5]. In this article, we will introduce a new digital tool that may help painters who have not yet fully mastered the complexities of colour mixing. This tool may become helpful in those cases where a painter has a clear idea about the colour he/she likes to accomplish on the canvas. In such cases, inspiration may come from a piece of material, a cloth or a flower, but creating exactly that colour by mixing pigment paints on the palette may require perseverance and substantial time.

Since the 1960s it is possible to let computers calculate the optimum paint mixture. This approach needs a spectrophotometer that measures reflectance values as a function of wavelength, and requires colour formulation software to find the best paint mixture. Over the past decades, mixing machines in do-it-yourself shops have proven that this approach is technically feasible. However, for painters this approach is too costly, since the price of spectrophotometers is (far) beyond €5000.

In the past decade, for the first time, colour instruments have become available on the market at prices below €300 [6]. In this article, we investigate the potential of these instruments to guide artists on mixing pigment paints in order to achieve a desired colour. For this we chose the Spectro1 instrument (Variable inc., Chattanooga - USA). This is the first low-cost colour instrument that produces spectral data similar to any other spectrophotometer.

This paper is organised as follows. First, the Spectro1 instrument is described, along with the line of oil paints that we used as an example in this investigation. We present the process of determining spectral properties of the oil paints, and the optical model that we used to calculate the optimum mixture of pigment paints. These calculations use colour measurements as a starting point. Finally, we present and analyse the results of this approach for 36 different target colours, taken from reproductions of art works and common objects. The paper ends with conclusions.

Instrument and paint samples

Description of the instrument

Since 2005, ten different colour instruments became commercially available with prices below €300 [6]. These low-cost colour devices directly measure values for three colour coordinates, and therefore they are colorimeters. This is in contrast to traditional instruments that measure reflectance for a series of wavelength values. These are the so-called spectrophotometers, which are much more expensive than colorimeters. Unfortunately, colorimeters are not useful for calculating paint mixtures, since the three coordinate values that they measure are not sufficient to calculate concentrations for a larger set of pigments. Also, the calculated mixtures may not have colours that are stable with changing spectral light conditions (colour constancy and metamerism).

Early 2019, the first low-cost colour instrument entered the market which does provide reflectance values for a range of wavelengths. Therefor this Spectro1 instrument is positioned by its vendor Variable as the first low cost spectrophotometer. It is based on sensor technology from AMS (Premstaetten, Austria) that provides spectral information for eight different optical wavelengths [7], combined with smart technology to produce reflectance data across the visual spectrum at 10 nm resolution. The design produces a measurement geometry that has similarities to traditional diffuse geometry (specular excluded) and 45/0 geometry.

Because the instrument provides spectral data, we test if this instrument can be successfully used for calculating pigment mixtures of artist oil paints, as a digital and objective method to obtain any desired colour.

Paint samples

For this investigation, we used a selection of twelve pigmented oil paints from product line Van Gogh (Royal Talens, Apeldoorn - The Netherlands) [8]. In order to be able to formulate a large range of colours, the selection contained two different pigments for most colour categories (red, green, yellow and white), but three for the blue colour category and only one for black. The selection was partly inspired by the palette with thirteen pigmented oil paints that the artist Vincent Van Gogh used while staying in Arles, in Southern France (although Van Gogh used four different red pigments rather than two, and avoided using black pigment) [9]. Most of the artist's pigmented paints have a similarly named counterpart in the modern commercial paint line, although this obviously does not guarantee that the pigment constitution of the modern paint line is the same or even similar to the historic palette. Indeed, some pigments used by the artist have no counterparts in the modern palette at all. There is no counterpart for Vincent Van Gogh's Geranium (Eosin) Red pigment paint. This pigment was not replaced in the current investigation since we already had included two other red pigments. Vincent Van Gogh's Chrome Yellow Lemon and Chrome Yellow are no longer available because of their toxicity. Therefore we replaced them by modern Azo Yellow Lemon and Azo Yellow Medium pigments. For the blue and green ranges, we included the counterparts of all pigments that Vincent Van Gogh used.

The mixing behaviour of pigmented oil paints is not determined only by their colour, but their spectral properties need to be taken into account as well [3]. In order to determine the colour mixing properties of each chromatic pigmented oil paint, these were mixed in standardised ratios with white in approximately 50:50 and 10:90 mixtures, as well as with black in 1:99 and 3:97 mixtures. For achromatic pigmented oil paints, an adapted scheme was used. These schemes originate from their use in the paint industry as described in an earlier publication [10].

While weighing and mixing, some small deviations from this scheme appeared. This was not problematic, since we carefully weighed and registered all concentrations that we actually used. These actual mixing ratios are presented in Tables 1 and 2.

	Pure	First Titanium White	Second Titanium White	First Ivory Black	Second Ivory Black
Carmine Red (PR83/PR57:1)	100%	50.2%	9.95% [1])	99.01%	97.09%
Vermilion Red (PO34/PR57:1)	100%	50.00%	9.85%	99.01%	97.10%
Viridian Green (PG7/PY43)	100%	50.00%	11.47% [1])	99.01%	97.09%
Emerald Green (PW6/PB15/PY3)	100%	48.75%	9.96%	99.01%	97.09%
Ultramarine Blue /Synthetic (PB29)	100%	49.95%	9.73% [1])	99.01%	97.18%
Cobalt Blue (PB28)	100%	49.90%	11.45%	99.01%	97.18%
Prussian Blue (PB27)	100%	49.90%	9.91%	99.01%	[2])
Azo yellow Lemon (PY3/PW6)	100%	49.85%	10.01%	99.01%	96.90%
Azo Yellow Medium (PY74/PW6)	100%	50.10%	10.03% 7.48%	99.01%	97.09%

Table 1: Mixtures as prepared to characterise the colour properties of chromatic pigmented paints. [1])These mixtures also contribute to the characterisation of Titanium White. [2])Special mixture: 80.48% Titanium White + 10.06% Ivory Black + 9.46% Prussian Blue.

	Pure	Mixtures Ivory Black	Carmine Red	Viridian Green	Cobalt Blue
Titanium White saffl. oil (PW6/PW4)	100%	49.75%, 80.10%, 90.00%, 97.37%, 99.01%	90.05% [3])	88.53% [3])	88.55% [3])
Zinc White (PW4)	100%	79.68%, 91.31%, 99.01%	90.93% [3])	89.69% [3])	90.70% [3])
Ivory Black (PBk9)	100%		0.99%[3]), 2.91% [3])	0.99%[3]), 2.91% [3])	0.99%[3]), 2.82% [3])

Table 2: Similar as Table 1, for characterising achromatic pigmented paints. [3])These mixtures also contribute to characterising the chromatic pigments.

All paints were weighed with a digital balance with 0.01 gram accuracy according to its technical specifications. Since we are interested in testing a procedure that should be financially feasible for amateur and professional painters, we used a consumer balance from vendor Lenv which is available for less than 20 euros. We used the same balance to also determine the weight of the paint mixture that was applied, including corrections for paint remains in the brushes after application. We measured the surface area covered by each paint mixture and the volumetric mass density of each pigmented paint. By combining these measurement data we were able to estimate the thickness of the paint layer, assuming a uniform paint film thickness. Although this assumption is probably not very accurate, it does not affect the main results and conclusions from this investigation since almost all mixtures were found to be opaque. The four only exceptions are discussed further below.

All mixtures were applied on a white canvas, that we had covered by black stripes of pure Ivory black paint. Each mixture was applied partly on the white and partly on the black substrate in order to assess the transparency of each mixture. This is shown in Figures 1 and 2.

In a few cases, we found that the paint was not opaque, as illustrated on the right hand side of Figure 2. These were the pure (100%) mixtures of Titanium white, Zinc white, Azo yellow lemon and Azo yellow medium. For each of these four cases, we created three more applications with different film thickness. This makes it possible to also predict transparency and opaqueness of paint mixtures.

Figure 1: Photograph of 40 paint mixtures as applied on white canvas with black stripes as substrate and purple wall as background.

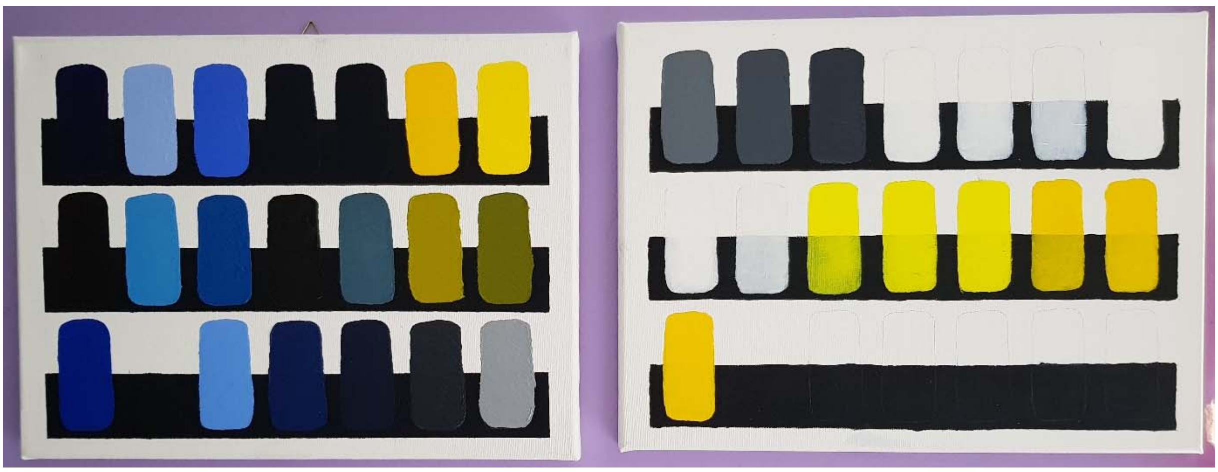

Figure 2: Photograph showing another 20 paint mixtures on the left. On the right, 15 other mixtures are shown with varying film thickness, leading to different levels of transparency.

Theory and process to determine parameter values

In this section, we will describe the step by step calculation procedure and the physical-mathematical model that we used to convert colour measurement values of a target colour into a paint recipe, showing which pigments should be used in what quantities in order to create the target colour with an oil paint mixture. Readers who are interested in our results, but much less in the procedure we followed to obtain them, may consider skipping this section. Readers who are interested in more details than those presented here can find them in references [11-13, 14 p381-403, 15 p102].

The non-hiding Kubelka Munk model

In this investigation we use the non-hiding form of the Kubelka-Munk theory [15-16]. In this theory, two parameters determine the optical properties of a paint. These are the absorption K and the scattering S, both parameters being a function of wavelength. The theoretical reflectance R_t of the paint is calculated by [17]:

$$R_t = \frac{(a+b)(a-b-R_g)\exp(-2bSD)-(a+b-R_g)(a-b)}{(a-b-R_g)\exp(-2bSD)-(a+b-R_g)} \tag{1a}$$

$$a = 1+\frac{K}{S} \tag{1b}$$

$$b = \sqrt{a^2-1} \tag{1c}$$

Here, R_g is the reflectance of the substrate under the paint, which is either the white of the canvas or the black of the Ivory black stripes. We note that equation (1) makes it possible to determine the opacity of every oil paint, and the influence of paint thickness. Further, R_t is the theoretical reflectance of the paint and D is the paint film thickness. Since instruments measure paint reflectance R_m outside the paint layer, we need to account for light reflection at the paint-air interface using the Saunderson correction [15].

$$R_m = \alpha k_1 + \frac{(1-k_1)(1-k_2)R_t}{1-k_2R_t} \tag{2}$$

Since the Spectro1 instrument has a lighting geometry similar to both diffuse specular excluded geometry and 45/0 geometry, we use parameter values $k_1 = 0.04$ and $k_2 = 0.49$, $\alpha = 0$ [15]. The reflection of paint mixtures is described by the well-known relationship proposed originally by Duncan.

$$\left(\frac{K}{S}\right)_{mixture} = \frac{c_1 K_1 + c_2 K_2 + \ldots + c_N K_N}{c_1 S_1 + c_2 S_2 + \ldots + c_N S_N} \quad (3)$$

where N is the total number of pigmented paints in the mixture, c_i is the weight concentration of pigmented paint number i, and K_i and S_i are the Kubelka-Munk parameters for pigmented paint number i [16, 18]. Since the weight concentrations ci and paint film thickness D were measured for each pigment paint mixture, we were able to find numerical values for the Kubelka-Munk K_i and S_i parameters for each pigmented paint in such away that the reflectance values predicted by the model best fit the values measured by the Spectro1 instrument.

Process for predicting oil paint mixtures

Once the values for the Kubelka-Munk K_i and S_i parameters were determined for each of the 12 different pigmented oil paints, we could use the same equations (1) up to (3) to predict which paint mixture best matches a measured colour. As an example, we used the Spectro1 instrument to measure the reflectance curve Ractual of the red patches from a poster reproduction of the well-known painting Victory Boogie Woogie (1942-1944) by the Dutch painter Piet Mondriaan.

With a spreadsheet program we optimised the weight concentrations c_i of each of the pigmented paints listed in Tables 1 and 2 using the following procedure. We first minimised the absolute difference between predicted reflectance Rm and actually measured value Ractual, after summing over all wavelengths. In the next step, we minimised the colour difference ΔE between measured and predicted reflectance. Here, we use the CIEDE2000 (ΔE_{00}) equation with (1:1:1) parameters, 10° standard observer and D65 illuminant (here, we did not take into account metamerism, although the current analysis could be easily extended to do so) [19]. In the third step, we reduced the number of pigmented paint components in a mixture without allowing the colour difference become too large. Values of ΔE_{00} < 0.5 were considered acceptable. In this way we aimed at obtaining theoretically acceptable colour matches by mixing a maximum of five oil paints.

Experimental

We measured reflectance from the 36 different objects listed in Table 3. The objects were selected as representing sources of possible colour inspiration. They could be common household objects such as furniture, flowers, home appliances, plastic plates or wall colours.

We also used colours taken from two paintings. We first used a reproduction from the well-known Yellow House (1888) by Van Gogh. As with most of the works from this artist, its colours are expressed as dramatic brush strokes. We limited ourselves to the sky area in this art work, and measured a range of four different blue shades.

The second art work that we included in this investigation is Victory Boogie Woogie (1942-1944) by Piet Mondriaan. In this painting, the artist applied colours uniformly which makes colour measurement easier. We measured reflectance at 11 different spots for this painting.

For all 36 colour samples, we followed the procedure outlined in the previous section to predict which mixture of pigmented oil paints best matches the sample.

Sample number	Description	Category	Colour group	No. of paints	ΔE_{00} (target-paint)	ΔE_{00} (target-predicted)	ΔE_{00} (paint-predicted)
1	Refrigerator	HA	Light purple	4	2.18	0.00	2.22
2	Wall	W	White	3	4.42	0.72	4.35
3	Wall	W	Greenish blue	4	2.45	0.00	2.45
4	Wall	W	Greenish blue	5	1.86	0.82	1.19
5	Wall	W	Greenish blue	4	3.29	0.00	3.30
6	Wall	W	Greenish blue	5	1.79	0.46	1.58
7	Bathroom tile	W	Light blue	4	2.70	0.00	2.70
8	Bathroom tile	W	White	4	1.79	0.00	1.79
9	Bathroom tile	W	Dark grey	4	1.46	0.00	1.44
10	Table	FU	Dark grey	4	3.51	0.00	3.50
11	Table	FU	White	3	1.08	0.00	1.05
12	Table	FU	Deep red	3	5.35	4.08	1.41
13	Vase	H	Dark red	3	5.17	5.13	1.07
14	Plastic plate	H	Light green	4	2.01	0.00	2.02
15	Plastic plate	H	Deep purple	3	1.86	2.75	0.91
16	Plastic plate	H	Light blue	3	1.48	0.69	1.04
17	Reproduction Yellow House (Van Gogh)	PR	Blue sky	5	2.29	0.14	2.38
18	Reproduction Yellow House (Van Gogh)	PR	Blue sky	3	5.50	0.69	5.55
19	Reproduction Yellow House (Van Gogh)	PR	Blue sky	3	3.00	0.11	2.96
20	Reproduction Yellow House (Van Gogh)	PR	Blue sky	3	1.00	0.87	1.52
21	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Red	4	1.06	0.53	1.09
22	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Yellow	4	2.35	0.47	2.17
23	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Blue	4	3.10	0.00	3.11
24	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Mid-grey	4	0.41	0.04	0.38
25	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Nearly white	4	2.04	0.00	1.99
26	Reproduction Victory Boogie Woogie (Mondriaan)	PR	White	5	0.48	0.00	0.53
27	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Black	4	4.04	0.00	4.02
28	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Black	1	11.52	11.1	2.26
29	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Dark grey	5	3.32	0.00	3.26
30	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Dark grey	4	2.83	0.00	2.79
31	Reproduction Victory Boogie Woogie (Mondriaan)	PR	Orange yellow	6	2.51	0.43	2.35
32	Flower (Gerbera)	FL	Orange	3	5.24	0.22	1.11
33	Flower (Gerbera)	FL	Pink	4	1.40	0.00	1.94
34	Flower (Gerbera)	FL	Red	3	12.82	1.53	1.89
35	Flower (Gerbera)	FL	White	5	2.62	0.00	2.34
36	Flower (Gerbera)	FL	Green	4	5.05	0.00	4.34

Table 3: Samples selected for reflectance measurement and colour reproduction in oil paint. Samples are taken from the following categories: Home appliances (HA), Wall colours (W), Furniture (FU), Home (H), Paint reproductions (PR) and Flower (FL).

Results

We continued our investigation by preparing all 36 proposed mixtures with the oil paints, and measuring the reflectance values for each paint sample. In table 3, the column headed by ΔE_{00} (target-paint) shows the colour difference between measured reflectance of target colour and measured reflectance of paint mixture. For the 36 target colours we find a median value of ΔE_{00} = 2.48 (average: 3.19). This colour difference is the combined result of different sources of error: (i) errors in the optical model, i.e. the accuracy of the predictions, (ii) measurement errors by the instrument, (iii) colour variations of the paint itself, (iv) the impossibility to reach some target colours (also taking differences in opacity into account) by using a limited set of 12 different pigments, (v) the impossibility to reach some target colours by using a maximum of 5 components in a pigment mixture. We note that even for mixtures that did need 5 components, usually 2 or 3 of the components needed to be added in very little amounts. Therefore the colours of these mixtures do not become dull.

The last two columns of table 3 quantify contributions of prediction error and measurement error to the total colour difference ΔE_{00} (target-paint). The column headed by ΔE_{00} (target-predicted) presents colour differences between the measured target colour and the theoretical reflectance that we predict for the optimised mixtures. The table shows that for 25 out of the 36 selected colours, we were able to find a theoretically acceptable match with $\Delta E_{00} < 0.5$ by mixing at most 5 from the 12 pigmented paints. In 18 from these cases, the predicted colour difference between the measured reflectance and the proposed mixture is ΔE_{00} (target-predicted) = 0.00. The average colour difference that is predicted for the 36 selected colours is $\Delta E_{00} = 0.86$. For predicting colours of artist paints, this performance can be considered to be "quite well" [20].

We further analyse the 11 colour samples for which no theoretically acceptable match had been found. In four cases (sample numbers 2, 4, 20 and 21) we are able to find a theoretically acceptable colour match if we allow more than 5 components in the mixture. In the other seven cases, the calculations indicate that the colour cannot be reproduced by the selected set of pigmented oil paints. In some cases, the impossibility to obtain a perfect colour match is due to the fact that the sample colour has a lower gloss level than the oil paints we use here (in this investigation, we did not attempt to modify the gloss level of the oil paint by adding medium or varnish). This is the case for the colours of the flower (numbers 32 and 34). The black colour used by Piet Mondriaan (sample 28 in Table 3) is also impossible to match, because it is darker than the darkest pigment paint (Ivory black) that we selected for this investigation. For a painter, it can be very helpful to know that obtaining a particular colour can only be achieved by adding a different pigmented oil paint to the palette.

Visually, almost all paint mixtures show an excellent colour match with the colour of the original object. This is illustrated by a number of representative photographs in Figures 3 and 4. In all these cases the results indicate that the proposed method satisfies the objective that we set when starting this investigation, i.e. providing an instrumentally based method to support painters in obtaining intended colours by mixing colours.

(a) (b)

(c) (d)

Figure 3: Photographs of dry paint mixtures. Their colours are compared to the colours of the original objects placed next to them: (a) Table grey and red samples 10 and 12, (b) Plastic plates samples 14, 15 and 16, (c) Bathroom tile samples 7, 8 and 9, (d) Blue sky in Van Gogh painting of Yellow House, samples 17-20.

(a) (b)

Figure 4: Photographs of dry paint mixtures. Their colours are compared to the colours of the original objects placed next to them: (a) Wall paint sample 3, (b) Various colours in Mondriaan painting Victoria Boogie Woogie, samples 21-31.

The visually acceptable colour matches shown in Figures 3 and 4 are a clear indication that the proposed process for instrumental colour matching could be useful for painters. In order to further quantify the accuracy of this method, we prefer not to only use the values of colour differences ΔE_{00}(target-predicted) in table 3, because these values are strongly affected by the (im)possibility if target colours can be matched by the given set of oil paints, and also by our constraint to mix only five oil paints or less. Therefore, for each mixture we also calculated the colour difference ΔE_{00}(paint-predicted) between the predicted reflectance curve and the measured values after applying the paint mixture.

The last column in Table 3 shows that for many mixtures the values of ΔE_{00}(paint-predicted) are larger than the theoretical value ΔE_{00}(target-predicted). We find an average value ΔE_{00}(paint-predicted) = 2.22 (median value: 2.09). This will appear as a small but visible colour difference to most painters. This means that the colour recipes that are predicted are an excellent starting point for painters, even though they are often not an exact colour match.

The magnitude of the average error ΔE_{00}(paint-predicted) = 2.22 is partly caused by imperfections of the Kubelka-Munk model. Model errors can be counteracted by calculating corrections to the calculated mixtures (see e.g. [21] and [14 p427]). However, the error that we find here is larger than values for the model error mentioned in academic and industrial research, such as ΔE_{00} = 1.0 [20, 22] and ΔE_{ab} = 0.5 [14 p403]. Therefore, in the final section we will test if the error found here is caused primarily by the simple tools that we used for weighing, mixing and application.

Discussion on error

After measuring all 36 samples, we repeated the complete series of measurements. In this way we find a repeatability of measurements equal to ΔE_{00} = 0.23 on average (median value: 0.20, maximum: 0.51). In this series, the 90% percentile is given by ΔE_{00} = 0.34. These numbers show that repeatability errors of the instrument are so small that they have hardly any impact on the quality of the colour match.

We selected 5 from the 36 paint mixtures to investigate the repeatability of the mixing and application process. Each mixture was mixed and applied independently of processing the first series of 36 mixtures. After measuring these five paint mixtures, we find that the colour difference between the first and the second application is ΔE_{00} = 1.49 on average (median value: 1.16, maximum: 2.60). We conclude that the variability errors from mixing and application do have a large impact on the overall colour error.

We find the largest errors for those mixtures that contain 0.02 grams or less of pigment Ivory Black as part of a 10 grams mixture. Since this pigment has a very large colour strength, small dosing errors lead to relatively large colour differences. Indeed, the colour differences that we found are mainly caused by lightness differences, and they may therefore have been caused by slightly varying amounts of Ivory black pigment. We had already tried to avoid this type of errors by preparing pre-mixtures of this pigment with one of the other constituent pigments. This was necessary also because the weighing scale that we used has a smallest measurement unit of 0.01 grams, making it very challenging to weigh 0.02 gram or less. The current results indicate that this pre-mixing strategy was not completely successful.

Summary

We investigated new digital technology that may guide painters in mixing oil paint colours to achieve any colour. This approach is based on a new instrument that is able to measure colour and that is available at a price affordable to many artist painters. We used this Spectro 1 instrument to colour match 36 different object colours with a commercial line of artist oil paints (Royal Talens - Van Gogh). All colours were theoretically matched using a simple spreadsheet program, and the optimised paint mixtures were prepared and applied on a canvas. We measured the resulting colours, and compared them with the targeted object colours.

The investigation shows that most object colours could be matched quite well theoretically by mixing 5 or less different oil paints from a preselected set of twelve. This resulted in an average value of ΔE_{00} = 0.86, indeed indicating that the quality of colour match is "quite good". After preparing and applying the predicted paint mixtures, the average colour difference increased to a value of ΔE_{00} = 2.22. We showed that the error is due mainly to (i) mixing errors, especially for mixtures containing very small quantities of black pigment, (ii) gloss differences between object colours and oil paint, (iii) limitations in colour gamut that are inherent in the selected set of oil paints, and to a lesser extend (iv) model errors.

References

1. Briggs D (2019), The dimensions of colour – http://www.huevaluechroma.com/011.php – last accessed 14 July 2019.
2. Berns RS (2014), Color Science for the Visual Arts, *Lecture notes of MSc Color Science*, Rochester Institute of Technology (USA), 88.
3. Briggs D (2018), Where is colour education now?, Munsell Centennial Color Symposium, Boston (USA). [http://www.iscc-archive.org/Munsell2018_Presentations/Briggs-Presentation-WhereIsColourEducationNow.pdf – last accessed 14 July 2019]
4. Edwards B (2004), *Color: A Course in Mastering the Art of Mixing Colors*, New York: Jeremy P Tarcher.
5. Braddy P (2012), Color mixing…You can't get it unless you do it!.[https://www.justpaint.org/color-mixing-you-cant-get-it-unless-you-do-it/ – last accessed 14 July 2019]
6. Kirchner E, Koeckhoven P and Sivakumar K (2019), Predicting the performance of low-cost color instruments for color identification, *Journal of the Optical Society of America A*, **36** (3) 368-376.
7. AMS (2019), New multi-channel spectral sensor from ams set to transform the market for mobile color and light measurement. [https://ams.com/-/new-multi-channel-spectral-sensor-from-ams-set-to-transform-the-market-for-mobile-color-and-light-measurement – last accessed 14 July 2019]

8. Royal Talens (2019), Colour chart Van Gogh oil colour. [https://www.royaltalens.com/media/4471873/Van-Gogh-oil-colour-ENG.pdf – last accessed 14 July 2019]
9. Geldof M, Megens L and Salvant J (2013), Van Gogh's palette in Arles, Saint-Rémy and Auvers-sur-Oise, in *Van Gogh's Studio Practice*, Vellekoop M, Geldof M, Hendriks E, Jansen L and de Tagle A (eds.), Brussels: Mercatorfonds, 238-254.
10. Geldof M, Ness Proano Gaibor A, Ligterink F, Hendriks E and Kirchner E (2018), Reconstructing Van Gogh's palette to determine the optical characteristics of his paints, *Heritage Science*, **6**:17, 1-20.
11. Berns RS (2009), *Billmeyer and Saltzmann's Principles of Color Technology*, Fourth Edition, Hoboken: Wiley, 176-181.
12. Nobbs JH (1997), Colour match prediction for pigmented materials, in *Colour Physics for Industry*, McDonald R (ed.), Bradford: Society of Dyers and Colourists, 292-372.
13. Berns RS and Mohammadi M (2007), Evaluating single- and two-constant Kubelka-Munk turbid media theory for instrumental-based in painting, *Studies in Conservation*, **52**, 299-314.
14. Klein GA (2010), *Industrial Color Physics*, New York: Springer.
15. Völz HG (2001), *Industrial Color Testing: Fundamentals and Techniques*, Second Edition, Weinheim: Wiley-VCH.
16. Grum F and Bartleson CJ (1980), *Optical Radiation Measurements: Volume 2 Color Measurement*, New York: Academic Press.
17. Kubelka P (1948), New contributions to the optics of intensely light-scattering materials: Part 1, *Journal of the Optical Society of America,* **38**, 448-457.
18. Duncan DR (1940), The color of pigment mixtures, *Proceedings of the Physical Society*, **52** (3), 390-401.
19. CIE (2001), Improvement to industrial colour-difference evaluation, *CIE Publication No. 142-2001*, Vienna: Central Bureau of the CIE.
20. Zhao Y and Berns RS (2006), Further investigations of colorant database development for two-constant Kubelka Munk theory for artist acrylic and oil paints, *Munsell Color Science Laboratory Report*.
21. McDonald R (1997), Recipe prediction for textiles, in *Colour Physics for Industry*, McDonald R (ed.), Bradford: Society of Dyers and Colourists, 346.
22. Gorji Bandpay M, Ameri F, Ansari K and Moradian S (2018), Mathematical and empirical evaluation of accuracy of the Kubelka–Munk model for color match prediction of opaque and translucent surface coatings, *Journal of Coatings Technology and Research*, **15** (5), 1117-1131.